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SHORT NOTE

Estimates of strain due to brittle faulting: sampling of fault populations

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Abstract—The geometry of sampling domains is a first-order consideration in the characterization of brittle fault populations. In most cases, descriptions of fault size distributions based on map, cross-section, traverse or borehole data systematically underestimate the number of small faults present in a volume. The geometry of sampling domains may be accounted for using an empirical proportionality between fault displacement and trace length. Estimates of strain in which the sampling geometry is considered suggest that small faults accommodate a significant portion of the total strain due to the brittle faulting process.

INTRODUCTION

A PROBLEM common to many studies of brittle fault populations is the quantification of sampling. A description of sampling is important to understand the reliability of both qualitative and quantitative analyses based on the data. We recently proposed a means of quantitatively assessing the sampling of brittle fault populations based on the conjecture that faults follow fractal (power-law) size distributions (Marrett & Allmendinger 1990). This approach is useful because only two parameters need be evaluated to construct a model with which to compare the data and quantify the portion of faults sampled in any size range. A valuable by-product of such a model is an estimate of the total strain due to faulting.

Since submission of the above cited work, we have become aware of data supporting fractal size distributions for brittle fault populations and analyses similar to our own for estimating the strain accommodated by a brittle fault population (Kakimi 1980, Villemin & Sunwoo 1987, Childs *et al.* 1990, Scholz & Cowie 1990). Here, we address two problems concerning these estimates. There is disagreement among various authors concerning the specific relationship between fault displacement and fault trace length or width (e.g. Walsh & Watterson 1988, Scholz & Cowie 1990), which is necessary to make strain estimates. A second problem concerns the geometry of the sampling domain in which a fault population is observed. Although the geometry of the sampling domain is of first-order importance for estimating strain, previous analyses have not taken this into account.

The geometrical dimensions (s) of the sampling configuration employed to study a brittle fault population commonly is lower than the geometrical dimension (f) of the region occupied by the fault population. Populations of faults that do not span the brittle layer of the Earth's crust occupy three-dimensional volumes ($f = 3$), but maps and cross-sections provide two-dimensional sampling ($s = 2$), and traverse and borehole data provide one-dimensional sampling ($s = 1$). Faults spanning the brittle crust occupy effectively two-dimensional shells ($f = 2$). Maps should provide complete two-dimensional sampling ($s = 2$), but cross-sections and traverse data provide equivalent one-dimensional sampling ($s = 1$), and borehole data provide zero-dimensional sampling ($s = 0$).

A sampling configuration with a geometrical dimension that is less than that of the region occupied by a fault population ($s < f$) results in underestimation of the number of small faults present in a region and, hence, of the contribution by small faults to the total strain accommodated by faulting. The reason for this is that a specific large fault is more likely than a specific small fault to be intersected by an arbitrary line or plane through the volume containing the fault population (e.g. Heffer & Bevan 1990). Thus, even if all faults are observed along a one- or two-dimensional sample of a three-dimensional fault population, the sampling of large faults will be more complete than the sampling of small faults. We consider this effect explicitly and show how to account for it in estimates of the total strain due to the entire fault population.

THEORETICAL BACKGROUND

The strain of a fault is directly related to the geometric moment (M_g) which may be expressed as the product of the average displacement ($\bar{d}$) and surface area (A):

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$$M_g = \bar{d} \cdot A. \quad (1)$$

Statistically, the average displacement is directly proportional to the maximum displacement or an arbitrary displacement measurement (d) (Marrett & Allmendinger 1990). Because three-dimensional control is rarely sufficient to evaluate the surface area of a fault, the trace length (l) on maps or cross-sections provides the most convenient measure of surface area. For faults that do and do not span the brittle crust, respectively:

$$M_g \sim d \cdot l \quad (2a)$$

$$M_g \sim d \cdot l^2. \quad (2b)$$

If fault populations follow fractal size distributions in terms of strain, then we expect that the cumulative number of faults (ΣN) with geometric moment greater than or equal to M_g can be represented by:

$$\Sigma N \sim M_g^{-B}, \quad (3)$$

where B characterizes the relative numbers of large and small faults (Marrett & Allmendinger 1990). If we further assume that d and l are systematically related, then we can write the following equations:

$$\Sigma N \sim d^{-C_1} \quad (4)$$

$$d \sim l^{C_2}. \quad (5)$$

Substitution of equations (2), (4) and (5) into equation (3) reveals for faults that do and do not span the brittle crust, respectively:

$$B = \frac{C_1 \cdot C_2}{C_2 + 1} \quad (6a)$$

$$B = \frac{C_1 \cdot C_2}{C_2 + 2}. \quad (6b)$$

Fault populations commonly are observed using sampling configurations having a lower geometrical dimension than that of the region occupied by the fault population. In the case of faults that do not span the brittle layer of the Earth's crust ($f = 3$), the probability of observing a specific fault in an arbitrary two-dimensional cross-section ($s = 2$) of the fault population depends directly on l. The same holds in all cases for which $f - s = 1$. The cumulative number of faults can thus be written:

$$\Sigma N' \sim \Sigma N \cdot l \sim d^{-C_1 + 1/C_2} \sim d^{-C_1'}, \quad (7)$$

where the primed variables are those of the lower-dimensional sample. It follows that:

$$C_1 = C_1' + \frac{1}{C_2}. \quad (8)$$

Similarly, the probability of observing a specific fault in a one-dimensional traverse ($s = 1$) across a population of faults that do not span the brittle crust ($f = 3$) depends on l^2. The same holds in all cases for which $f - s = 2$:

$$\Sigma N'' \sim \Sigma N \cdot l^2 \sim d^{-C_1 + 2/C_2} \sim d^{-C_1''}, \quad (9)$$

where the double primed variables are those of the sample. It follows that:

$$C_1 = C_1'' + \frac{2}{C_2}. \quad (10)$$

We wish to estimate the strain accommodated by a population of faults using information explicitly describing only the largest faults. For a fault population with an ideally fractal size distribution, the geometric moment of the Nth largest fault ($M_g^{(N)}$) can be expressed in terms of the geometric moment of the largest fault ($M_g^{(1)}$) and B:

$$M_g^{(N)} = M_g^{(1)} \cdot \frac{1}{N^{1/B}}. \quad (11)$$

The total geometric moment ($M_g^{(\text{total})}$) due to the combined effects of the N largest faults can then be written as follows (Marrett & Allmendinger 1990):

$$M_g^{(\text{total})} = M_g^{(1)} \cdot \left[\frac{1}{1^{1/B}} + \frac{1}{2^{1/B}} + \frac{1}{3^{1/B}} + \cdots + \frac{1}{N^{1/B}} \right]. \quad (12)$$

$M_g^{(\text{total})}$ converges only for $B < 1$, in which case the largest faults accommodate most of the strain. For $B > 1$, small faults accommodate more strain than large faults, so the size of the smallest fault must be known to make a useful strain estimate.

Scholz & Cowie (1990) integrated an expression similar to equation (11) for N from 1 to infinity, and obtained (simplifying their result using the present notation):

$$M_g^{(\text{total})} = \frac{B}{1 - B} M_g^{(1)}. \quad (13)$$

By numerically evaluating equation (12), we find that equation (13) underestimates $M_g^{(\text{total})}$. The reason for this is that the integral approach implicitly assumes N to be a continuous function, but N is defined to be integers. Stable estimates of $M_g^{(\text{total})}$ may be made by combining the summation and integral approaches using:

$$M_g^{(\text{total})} = M_g^{(1)} \cdot \left[\frac{1}{1^{1/B}} + \frac{1}{2^{1/B}} + \cdots + \frac{1}{N^{1/B}} + \frac{B(N+1)}{(1 - B)(N+1)^{1/B}} \right]. \quad (14)$$

For most values of $B < 1$, equation (14) converges rapidly as N is increased.

FAULT POPULATIONS

The hypothesis that fault populations follow fractal size distributions is supported by analysis of several data sets (e.g. Kakimi 1980, Villemin & Sunwoo 1987, Childs *et al.* 1990). These data have been used to make estimates of the total strain or extension accommodated by faulting (respectively by Kakimi 1980, Scholz & Cowie 1990); however the geometries of the sampling domains were not taken into consideration. Accounting for the sampling geometry may yield estimates of the net strain

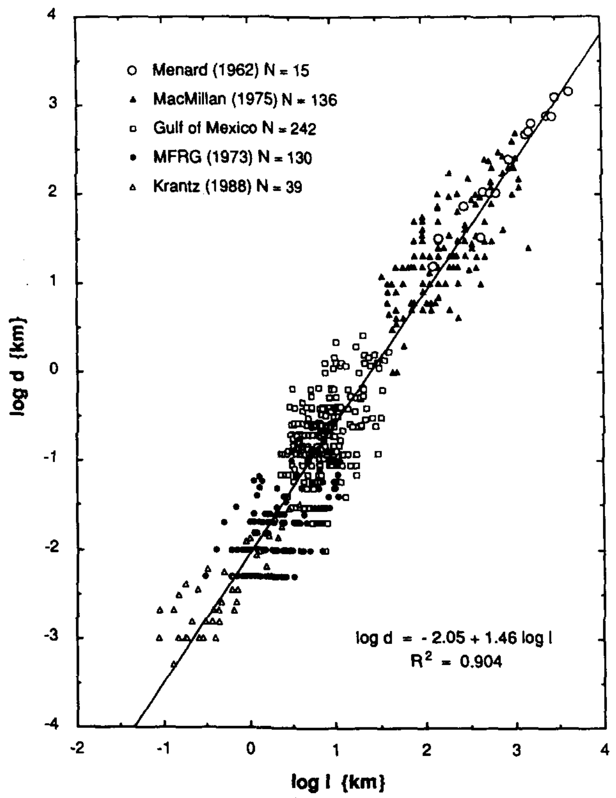

Fig. 1. Displacement–trace length data. The line represents a least-squares fit to all 562 data (R^2 is correlation coefficient).

due to faulting that are significantly greater than estimates that fail to account for the sampling geometry. We illustrate this using results from Childs *et al.* (1990) and an empirically based proportionality between fault displacement and trace length.

Displacement–trace length proportionality

The parameter C_2 (equation 5) was addressed by Walsh & Watterson (1988), who proposed a model for fault growth which predicts $C_2 = 2$. However, the least-squares best-fit to the data they analyzed was $C_2 = 1.58$. We compiled other data (Menard 1962, MacMillan 1975, MFRG 1973, Krantz 1988, Gulf of Mexico unpublished), none of which were used by Walsh & Watterson (1988), and find empirically that $C_2 = 1.46$ (Fig. 1). The fit to the data is quite good, considering that displacement varies over six orders of magnitude and trace length varies by nearly five orders of magnitude. However, the scatter also is quite large. For example, faults with 10 m of displacement have traces as short as 0.37 km and as long as 7.3 km (Fig. 1).

Both data sets presented here and in Walsh & Watterson (1988) suggest a value of C_2 near 1.5. Although this result disagrees with the fault growth model of Walsh & Watterson (1988), $C_2 = 1.5$ may be anticipated by making a slight modification to their model. In their model, the difference between slips during consecutive events on a fault is a constant, which results in $C_2 = 2$. If, instead, the difference between slips depends linearly on the total number of slip events that have occurred on the fault, $C_2 = 1.5$ results.

Scholz & Cowie (1990) also considered the parameter C_2 and argued that a value of about 1.0 is most consistent with data. However, with the exception of faults in Quaternary lacustrine strata, the data shown by Scholz & Cowie (1990) follow the same pattern as data presented here and in Walsh & Watterson (1988). Because the lacustrine strata are young and surficial, they probably are weaker than most brittle rocks (Walsh & Watterson 1988). This may explain why faults in the lacustrine strata have higher displacements than predicted by the fit to the data presented here (Fig. 1). The interpretation of Scholz & Cowie (1990) suggests, on the other hand, that the lacustrine strata are among the strongest materials containing faults they analyzed.

Size distributions of fault displacements

Reported values of C_1 (equation 4) represent faults that do not span the brittle crust and vary widely from 0.37 to 1.7 (Kakimi 1980, Villemin & Sunwoo 1987, Childs *et al.* 1990). We argue that a large part of the variation is an artifact resulting from different geometries of the sampling domains. Unfortunately, the sampling procedure is seldom described in studies of fault populations (e.g. Kakimi 1980, Villemin & Sunwoo 1987). Childs *et al.* (1990) present a number of displacement population analyses and carefully distinguish one- and two-dimensionally sampled data sets.

The values determined by Childs *et al.* (1990) for C_1' (1.0–1.7) are systematically higher than those determined for C_1'' (0.37–1.0). Furthermore, assuming $C_2 = 1.5$, note that the range of C_1' predicts the range of C_1'' (equations 8 and 10). Thus, the data are consistent with the proposed dependence of displacement population fractal dimension on sampling domain geometry, and the variation of the fractal dimension is much les than previously thought.

For the purpose of illustrating the importance of accounting for the geometry of the sampling domain of fault populations, we wish to determine a value for C_1. Most analyses of C_1'' are near 0.50 (Childs *et al.* 1990). Assuming $C_2 = 1.5$, we estimate $C_1 = 1.8$ (equation 10).

Estimation of total strain accommodated by faulting

Because strain depends directly on M_g, an estimate of the total strain accommodated by a fault population with an ideally fractal size distribution may be made knowing $M_g^{(1)}$ and B. In practice, a much more reliable estimate of total strain may be made by summing the moments of the largest faults studied and using the relationships presented here to account for smaller faults. Assuming the values determined above of $C_1 = 1.8$ and $C_2 = 1.5$, we may calculate $B = 0.77$ (equation 6) and $M_g^{(\text{total})} = 4.0\, M_g^{(1)}$ (equation 14). If the sampling geometry had not been accounted for and $C_1 = C_1'' = 0.50$ were assumed, we would calculate instead $B = 0.21$ and $M_g^{(\text{total})} = 1.04\, M_g^{(1)}$.

In both cases $M_g^{(1)}$ is the same, so the strain estimates differ by a factor of about 4. By assuming larger values for C_1'' in the calculations of the previous two paragraphs, strain estimates will differ by even greater factors. The differences between strain estimates that do and do not account for the geometry of the sampling domain reflect the importance of small faults in accommodating strain. Thus, accounting for the geometry of the sampling domain is of first-order importance for estimating strain from fault data.

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